Empirical modelling of the thermal generation cost function for the Brazilian hydrothermal scheduling problem

Modelagem empírica da função de custo de geração térmica para o problema do despacho hidrotérmico brasileiro

Deisy Avila Morales
Marcelo R. Bessa
Daniel H. Marco Detzel

Universidade Federal do Paraná
deisyavilamo@gmail.com
marcelo.r.bessa@gmail.com
daniel@lactec.org.br

Abstract: The scheduling problem of a hydrothermal electric system is very complex, since its modelling involves a large number of variables and constraints that, in many cases, are simplified to make it feasible in computational terms. These simplifications imply the adoption of approximate models, which usually do not represent a realistic behaviour of the power plants and may interfere with the process of decision making of economic dispatch. This paper proposes an empirical model that defines the costs of gas-fired power plants, in order to represent the thermal dispatch in a more realistic fashion. This model is also suitable for nonlinear programming for the hydrothermal scheduling problem. In this sense, the suggested function penalizes start/stop operations and low power orders. The performance of the function is evaluated using the PHOENIX hydrothermal scheduling optimization model, through the simulation of a reduced test system of the southern region of Brazil. For comparison, the same system is optimized considering a traditional quadratic function to represent the thermal operational costs. Results revealed a satisfactory behaviour of the proposed function in the optimization model, allowing a decrease in thermal dispatches in low power ranges, which negatively impact generation and environmental costs. It was observed that the function was able to keep the thermal power plant off for longer periods of time than the conventional quadratic model and to reduce by 72% the number of dispatches below the 70% of the plant's installed capacity. A reduction in the processing time of about 32 minutes stands out, implying improvements in the computational performance of the model.

Keywords: hydrothermal electrical systems, economic dispatch, gas-fired power plant, thermal dispatch, nonlinear optimization.

Resumo: O planejamento da operação de um sistema elétrico hidrotérmico é um problema complexo, uma vez que sua modelagem envolve um grande número de variáveis e restrições que em muitos casos são simplificadas para torná-la viável computacionalmente. Essas simplificações implicam modelos aproximados, que usualmente não representam o comportamento real das usinas e que interferem na tomada de decisão do despacho econômico. Este artigo propõe uma modelagem empírica que define os custos das unidades termelétricas a gás, a fim de representar o despacho térmico de maneira mais realista e que seja adequada à programação não linear para o problema do despacho hidrotérmico. Nesse sentido, é sugerida uma função que leva em consideração uma penalização devida a custos de partida e custos ambientais para despachos de baixa potência. O desempenho da função é avaliado no modelo de otimização para o despacho hidrotérmico PHOENIX, mediante a simulação de um sistema teste reduzido da região Sul do Brasil. Os resultados revelaram um comportamento satisfatório da função no modelo de otimização, permitindo a diminuição nos despachos térmicos em faixas de baixa potência, os quais impactam negativamente nos custos de geração e ambientais. Observou-se que a função conseguiu manter por mais tempo a usina térmica desligada e reduzir em 72% o número de despachos abaixo de 70% da potência da usina, quando comparada com a função atualmente utilizada pelo modelo. Destaca-se uma redução no tempo de processamento de cerca de 32 minutos, implicando melhoras em relação ao desempenho computacional do modelo.

Palavras-Chave: sistema elétrico hidrotérmico, despacho econômico, termelétricas a gás, otimização não linear.

1 Introduction

The operation of a hydrothermal electric generation system has the objective of supplying electric energy through the appropriate management of available resources in the most economical way possible, i.e., minimizing the costs of the energy generated by a thermoelectric power station and possible energy deficits. This operation depends on the available energy that is limited, basically, by the storage capacity of reservoirs.

Decisions made in the present inevitably affect the system's future operation. If one uses hydroelectric energy now (depleting the reservoirs) and a drought period occurs in the future, it will be necessary to meet the energy demand with

higher energy generation costs. In this situation, there is also a higher risk of not fulfilling the energy demand. Contrarily, if the thermoelectric power plants are used now (keeping the reservoirs full) and a period of high inflows occurs in the future, spills will happen, thus causing a waste of energy and increasing the total cost of operation [1].

The planning of the operation of large hydrothermal systems, such as that of Brazil, is a complex technical task from the mathematical point of view due to aspects such as: the stochastic nature of inflows, the limited capacity of the reservoirs, the operational interdependence in the hydroelectric power plants disposed in series, the nonlinearities of the thermal power plants costs functions, and hydroelectric generation functions [2]. To obtain a viable solution to the optimization problem, simplifications are considered in short, medium and long-term planning horizons. These simplifications imply different approaches and, consequently, different modelling of the problem.

The Brazilian National Interconnected System planning and operation is performed by the Brazilian National Electrical System Operator (ONS), using the models developed by "Centro de Pesquisas de Energia Elétrica" (CEPEL) (free translation: Electric Energy Research Centre), a research centre owned by Eletrobras. The medium-term planning model NEWAVE [3], determines the operative decision through a simplified representation of the generation power plants considering equivalent energy systems. However, it cannot be guaranteed that the generation target of the equivalent systems will be viable when considering the plants individually. In order to define the operational policy in the short term, the DECOMP model [1] is used, allowing the determination of the individual generation amounts of each plant considering the characteristics and limitations associated with the operation. These models are based on the Stochastic Dual Dynamic Programming (PDDE) method [4] [5].

The call for proposals no. 001/2008 of the Research and Technological Development Programme of the Brazilian Electric Energy Sector regulated by "Agência Nacional de Energia Elétrica" (ANEEL) (free translation: Brazilian National Electric Energy Agency), gave an opportunity to explore alternative models comprising new approaches and innovative contributions. In this context, the project entitled "Optimization of hydrothermal dispatch through hybrid algorithms with high performance computation" (PHOENIX) [6] was developed. The proposed model consists of using techniques based on implicit nonlinear stochastic optimization such as the Augmented Lagrangian and the Spectral Projected Gradient methods, using an objective function that explicitly considers the total cost of operation simultaneously with the risk of energy deficits. Unlike the current approach, the project proposed the individual modelling of hydropower and thermal power plants.

A fundamental variable for decision making in hydrothermal optimization models is the thermal generation cost. In Brazil, medium- and long-term energy dispatch optimization models often consider a simplified approach in defining the thermoelectric generation costs. It does not reflect the actual behaviour of thermal power plants since relevant operating restrictions of the units are disregarded.

In this regard, Liu et al. [7] point that the cost function of a thermal generator is a combination of five terms: (i) energy generation cost, (ii) spinning reserve cost, (iii) startup cost, (iv) shutdown (either for protection system or manual intervention) cost, and (v) warm-keeping cost in sequence. The authors consider a detailed approach based on different operation stages for the plants. However, such level of detail is commonly addressed only in unit commitment problems.

In the literature, the thermoelectric scheduling problem is known as Dynamic Economic Dispatch problem [8]. Its objective is to solve the dispatch problem minimizing the thermal generation cost, which is based solely on fuel consumption. Recent studies, however, expanded the problem to consider environmental costs. The Dynamic Economic Emission Dispatch [9] [10] [11] [12] (or Combined Economic Emission Dispatch [13]) problem considers both the minimization of fuel costs and the emission of atmospheric pollutants in its formulation. Hence, it is modelled as a multi-objective optimization problem. Jebaraj et al. [13] assert that algorithm convergence and computational burden issues motivated the application of a wide variety of evolutionary algorithms, in detriment of classical mathematical approaches. On the other hand, Mahdi et al. [12] point out that conventional mathematical techniques may guarantee the solution optimality and that they are not dependent on heuristics or specific local parameters estimation.

In view of the aforementioned arguments, the present study aims at characterizing and modelling a function for the cost of generation related to gas-fired thermoelectric power plants. Such function should adequately represent underlying nonlinearities of the problem, and penalize low-power dispatches due to environmental and start/stop factors of the thermal units. Therefore, it offers alternatives such that it may overcome the present limitations of current nonlinear optimization models, when performing the operation of a hydrothermal dispatch.

2 PHOENIX model general formulation

The PHOENIX model [6] was proposed to solve the Brazilian medium-term hydrothermal scheduling problem. It minimizes the system overall operational costs by means of a nonlinear objective function. The problem is formulated as shown in Equation 1 [14],

$$\min \sum_{t=1}^{T} \lambda_t \left[\sum_{j=1}^{k} C_j\left(G_{T,j,t}\right) + \sum_{s=1}^{S} C_{D,s}\left(DEF_{s,t}\right) \right] \qquad (1)$$

where λ_t is the present value factor in time t, C_j is the thermal operational cost function for the thermoelectric plant j, $G_{T,j,t}$ is the thermal power generation for the thermoelectric plant j in time t, $C_{D,s}$ is the energy deficit cost function for the subsystem s, $DEF_{s,t}$ is the energy deficit for the subsystem s in time t, T is the considered time horizon, k is the number of thermal power plants in the system, and S is the number of subsystems considered. By default,

PHOENIX considers a 60 months optimization horizon, the same as the NEWAVE model [3]. On the other hand, the number of hydro and thermal power plants, as well as the number of subsystems, may vary depending on the selected system.

The problem is subjected to constraints such as water balance, energy demand fulfillment, and environmental flows. In addition, physical limitations for the individual hydropower plants (reservoir capacity, minimum and maximum discharge, and maximum power generation) and thermal power plants (minimum and maximum power generation) are considered. Finally, energy trade-offs among subsystems are also constrained.

To solve the hydrothermal scheduling problem, the Augmented Lagrangian and the Spectral Projected Gradient methods are applied. Such methods are known for their robustness and good convergence properties. For a detailed description of the methods, in conjunction with the aforementioned constraints of the problem, the reader is referred to the PhD thesis by Marcílio [14].

Considering the equation (1), this paper is focused on the function represented by $C_j(G_{T,j,t})$. The proposed model is described in the next section.

3 Modelling the cost of operation of thermal power plants

The modelling of a thermoelectric system is defined by the operating cost of the thermoelectric generating plants. Each individualized plant describes different characteristics that compose their total operational cost which should be considered, such as the type of fuel, minimum and maximum generation constraints, start and shutdown times, ramping rate, the efficiency of boilers and turbine/generator sets, environmental restrictions, etc. Therefore, the total fixed and variable cost of this system results is the sum of each individual unit cost. The main constraint of this system's operation is that the sum of the generated energy must equal the load requirement [15].

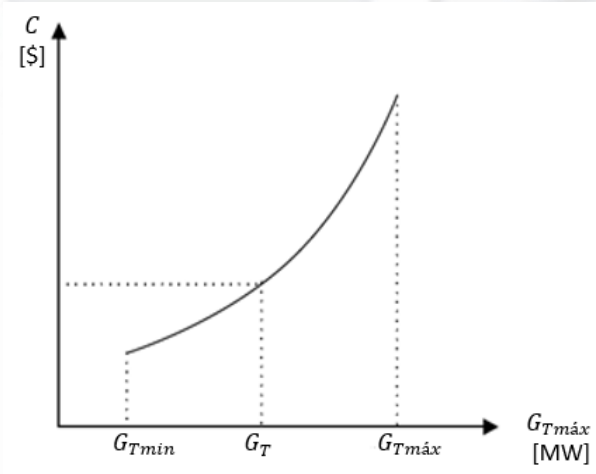

Figure 1: Typical cost curve of a thermal power plant.

The total cost of operation of thermal power plants includes variable and fixed portions. For most of the models, the fuel cost is considered variable, and the maintenance cost is considered fixed, as it is independent of the generation. This operational cost is usually modelled as a quadratic function [12] [16] [17] as shown in Figure 1. In the figure, the lower power limit generated $G_{T_{min}}$ represents the minimum economic generation. Below this value, the operation is technically and economically infeasible. Also, $G_{T_{max}}$ represents the uppermost limit, which coincides with the plant's maximum output power generation capacity.

The cost of operating a thermal plant is often expressed by Equation 2 [12] [16] [17],

$$C(G_T) = a_0 + a_1 G_T + a_2 G_T^2 \quad (2)$$

where C is the total operational cost of the thermal unit in the period, considered in monetary units $(\$/h)$; G_T is the power generated by the thermal unit in that time stage in (MW); and a_0, a_1 and a_2 are the coefficients of the production function of the unit, which is estimated by traditional least squares method or similar procedure for typical heat rates [16].

In the Brazilian context, the modelling is usually simplified by adopting the coefficients a_0 and a_2 as zero, making it a linear equation. The coefficient a_1 represents the fuel cost per MW of power generated and its value depends on the type of fuel used and the nominal capacity of the plant. The values a_1 for each of the thermoelectric power plants that integrate the National Interconnected System are available by ONS.

Medium-Term and Long-Term economic analyses generally admit that the thermoelectric power plants in the system are perfectly flexible. In this case, the economic dispatch of a thermal production system is defined based on quadratic costs as shown in Equation 2. This approach also assumes that the power plants operate at full load, within the maximum efficiency point and continuously over a period of time. However, this is not what happens in reality. Instead, there is a need to turn generating groups on and off for maintenance schedules (cycling operation), which causes the plant to operate at different load levels, i.e., outside their maximum efficiency point. The financial costs associated with starts/stops and cycling operations are very high [18].

When a thermal power plant operates at power levels other than the optimal, there is incomplete combustion and lower energy efficiency production. This requires the consumption of more fuel to produce the same amount of energy, potentiating the production of pollutant gases such as sulfur dioxide (SO_2), nitrogen oxides (NOx), carbon dioxide (CO_2). In gas-fired plants, the lowest CO_2 emission rates occur between 70% and 100% of the energy load [19].

Environmental regulations define the operating limits for combustion power plants to meet the allowed emission levels. The Brazilian legislation regulates the maximum emission limits for atmospheric pollutants from turbines through resolutions No. 436/2011 [20] and No. 016/2014 [21]. They establish that the emission limit for fixed power plants using gas turbines should be at the maximum those produced when it operates at 70% of its rated load.

Therefore, even without considering the costs caused by starts/stops, the increase in atmospheric emissions reinforces the need to avoid the dispatch of thermoelectric power plants at low power.

By using non-convex cost curves, it is possible to inhibit frequent start/stop operations and still encourage dispatches close to the optimum power of a thermal power plant. Based on environmental legislation, it is possible to define optimum ranges of operation above 70% of the nominal power of a thermoelectric power plant. The details of this approach are given in the next section.

4 Cost function estimation

In order to obtain a thermal generation cost function, this paper proposes a function that follows the format exhibited in Figure 2. In the generation range from 0% to approximately 70% of the power plant nominal capacity, the function presents a concavity and a local maximum. This peak represents the indirect costs due to the operation at low power levels and to the environmental costs. Therefore, it is intended for the optimizer to avoid operation decisions in this interval. In the range of 70% to 100%, the function assumes the typical quadratic cost form.

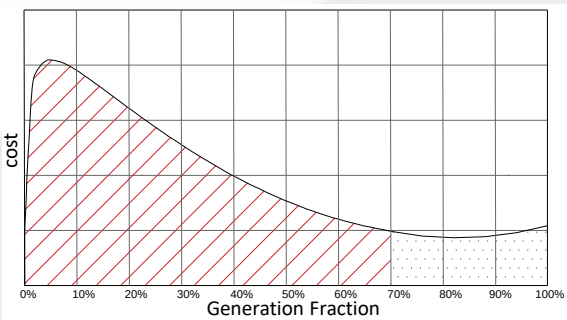

Figure 2: Proposed format for the thermal generation cost function modeling.

The methodology used to estimate such a function considered of an empirical adjustment using data obtained according to studies by Favoreto [22]. The author analysed the economic operation of a gas-fired power plant with an installed capacity of 500 MW, subject to the costs of fuel supply, transportation, commissioning, operation and maintenance (O&M), and gas supply contracts. The costs are summarized in Table 1.

To estimate a nonlinear cost function that can be used for all power plants, a modularized cost function is used, defined as a set of points whose cost C (image) and thermal generation G_T (domain), are within the interval [0, 1], or: $D(f) = \{G_T \in R | 0 \leq G_T \leq 1\}$, $Im(f) = \{C \in R | 0 \leq C \leq 1\}$.

One representation that fits the desired format is a function of the rational mixed type (Figure 3), defined by the ratio of a third-degree polynomial to a first-degree polynomial with four parameters, according to Equation 3.

$$C(G_T) = \frac{aG_T^3 + bG_T^2 + cG_T}{G_T + d} \quad (3)$$

The proposed model was adjusted to the cost and generation data of Table 1, by means of nonlinear least squares regression, using the MatLab® software. The adjustment process converged showing a high explanatory power, presenting a coefficient of determination R^2 above 99%. The model is differentiable throughout its entire domain, allowing derivation of the objective function in relation to the design variables. This is a required the criterion of the nonlinear optimization techniques [23].

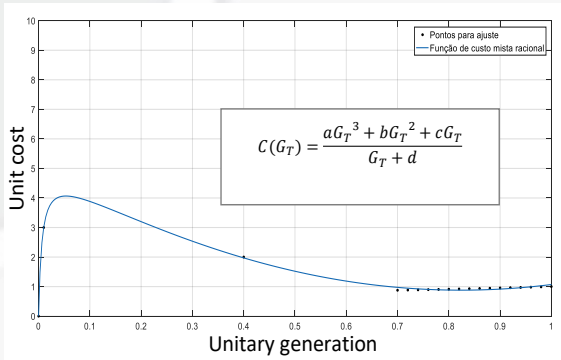

Figure 3: Suggested model for the cost of thermal generation.

5 Case study for model application

To evaluate the performance of the proposed thermal generation cost function, the PHOENIX optimization model was used [6] [14].

In this paper, a reduced test system was considered. It is based on actual data collected from the ONS and refers to power plants in the southern region of Brazil. It is composed of seven hydropower plants (Table 2) and one thermal power plant.

Table 1: Variable cost of a gas power plant, determined by simulation (Source: adapted from Favoreto [22]).

Operating time (%)	Variable cost (US/MWh)			Variable cost equivalent (R$/MWh)	Plant generation expected (MWh)	Equivalent total cost (R$)
	Minimum	Expected value	Maximum			
25	66,62	67,73	69,18	169,33	91.312,5	15.461.489,06
50		34,69	37,24	86,73	182.625,0	15.838.153,13
70	25,25			66,41	255.675,0	16.980.015,94
100		21,09	27,88	52,73	365.250,0	19.257.806,25

Table 2: Data of the hydroelectric power plants that integrate the test system.

Plant	Machadinho	Itá	Gov. Bento Munhoz	Salto Segredo	Salto Santiago	Salto Osório	Salto Caxias
Code	91	92	74	76	77	78	82
River	Uruguay	Uruguay	Iguaçu	Iguaçu	Iguaçu	Iguaçu	Iguaçu
Drainage area (Km²)	32050	44500	29900	34100	43330	45800	57000
Long-term average flow (m³/s)	728	1051	665	769	1020	1068	1371
Total reservoir volume (Km³)	3.3	5.1	5.8	2.9	6.8	1.1	3.6
Total head (m)	96.75	100.19	119.97	112.73	1020.9	66.17	62.99
Subsystem	1	1	1	2	2	2	2
Installed capacity (MW)	1140	1450	1676	1260	1420	1078	1240

Figure 4 shows the test system configuration. The streamflow time series for all hydropower plants range from January 1931 to December 2007.

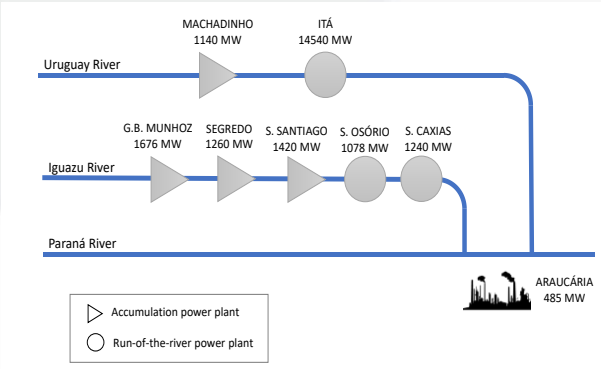

Figure 4: Reduced test system.

In this test system, a thermoelectric power plant was added to make it a hydrothermal system. This plant has 485 MW of installed capacity, and it is a natural gas-fuelled power plant. These characteristics were based on the Araucária thermal power plant, in the state of Paraná. The generation limits of this plant are defined in the interval between 0 and 433 MW/month.

Regarding the energy demand to be met, it is worth mentioning that ONS provides the demand curves only for aggregated subsystems. Hence, to obtain the demand for smaller systems, one may consider all the individual buses that are connected to the generation power plants. Moreover, information on energy trade-offs among neighbouring grids are not available. Therefore, to minimize the uncertainties of these processes, a flat energy demand estimated in 3262 MW/month was considered in this study. It is worth recalling that the main goal of this paper is to model the thermal power plant cost function in comparison with traditional approaches. In that sense, the definition of a realistic demand curve is of marginal concern.

6 Results and discussion

To validate the proposed model, 73 runs were performed in 60 months moving windows between 1931 and 2007. A comparative analysis of the results was made using the current cost function (quadratic cost function) and the proposed function (rational function) in three important aspects: thermal dispatch, hydrothermal system operation for three distinct hydrological scenarios, and computational time spent for running the model.

6.1 Thermal dispatch evaluation

The purpose of this analysis is to identify the operating features of the thermal power plant. Thus, they were classified into three generation intervals: the first when the plant is not operating, i.e., when the generation values of the power plant are equal to zero; the second when the plant generates above 0 up to 70% of its maximum capacity (303 MW/month), and the third when the plant generates more than 70% of the maximum capacity. Table 3 shows results in terms of the number of months that the plant was generating.

Table 3: Results of the thermal generation optimized with the quadratic function and the rational function.

Thermal generation classes (MW/month)	**Frequency quadratic function**		**Frequency rational function**	
	Nº of months	**%**	**Nº of months**	**%**
Equal to zero	663	16.3	1358	33.3
From 0 to 303	2130	52.2	863	21.2
From 303 to 433	1287	31.5	1859	45.5

Analysing the results shown in Table 3, the thermal dispatch presents an improvement with the inclusion of the rational function in comparison to the quadratic function, as evidenced by the decrease of the low-intensity operation. For the quadratic function, the power plant started 2130 times, corresponding to 52.2% of the total sample. In turn, with the rational function, it started 863 times or 21.2% of the total sample. This is equivalent to a reduction of 1267 times in the number of starts below 70% of the installed capacity mark.

Conversely, it can be observed that the number of starts above 70% of the installed capacity had an increase in relation to the quadratic function. With the rational function, the plant started 1859 times or 45.5% of the sample. For the quadratic function, it started 1287 times, which corresponds to 31.5% of the sample.

Figure 5 details the results dividing them into 11 ranges of generation intervals. A scale of 10% of the power plant

maximum capacity was considered. It should be noted that the first range corresponds to the times when the plant did not generate energy, i.e. the thermal generation was zero.

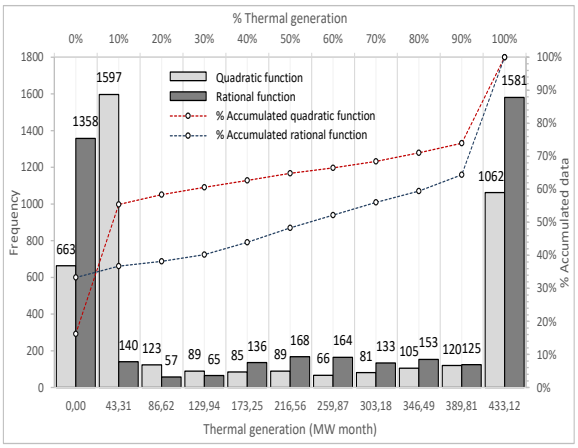

Figure 5: Histogram of thermal generation results for quadratic and rational functions.

According to Figure 5, a large part of the operation using the quadratic function occurs in very low power ranges, from 0 to 10% (between 0 and 43 MW/month). This does not happen in real situations since the operation of a thermal power plant in this range is not feasible either operationally or economically.

For this analysis, the main advantage of implementing the rational cost curve was that it kept the plant off more times. The model maintained the plant without generation over twice as long (1,358 months) when compared to the quadratic function (663 months). This directly impacts on the reduction of the operation cost.

6.2 Hydrothermal dispatch system for distinct hydrological scenarios

The results obtained for the hydrothermal dispatch generated by the PHOENIX model are presented in this section. To better understand the behaviour of thermal generation with other variables considered for the scheduling problem, results for two extreme hydrological scenarios (drought and wet periods) are shown. For comparison, results for an average streamflow scenario are also exhibited.

6.2.1 Drought scenario

Figure 6 shows the dispatch results referring to the low streamflow period between 1949 and 1953, both using the rational function and the quadratic function. As expected, the inflows sequences influence the thermal generation in both cases. However, the thermal dispatch increases with the use of the rational function; it is noted that the generations for months 9, 14, 18, 19, 24, 34, 48 and 49 present higher values and a more uniform trend compared to the quadratic function. In months 24 and 50, the quadratic function presented values of the thermal generation below 70%, while the rational function thermal dispatches remained as desired.

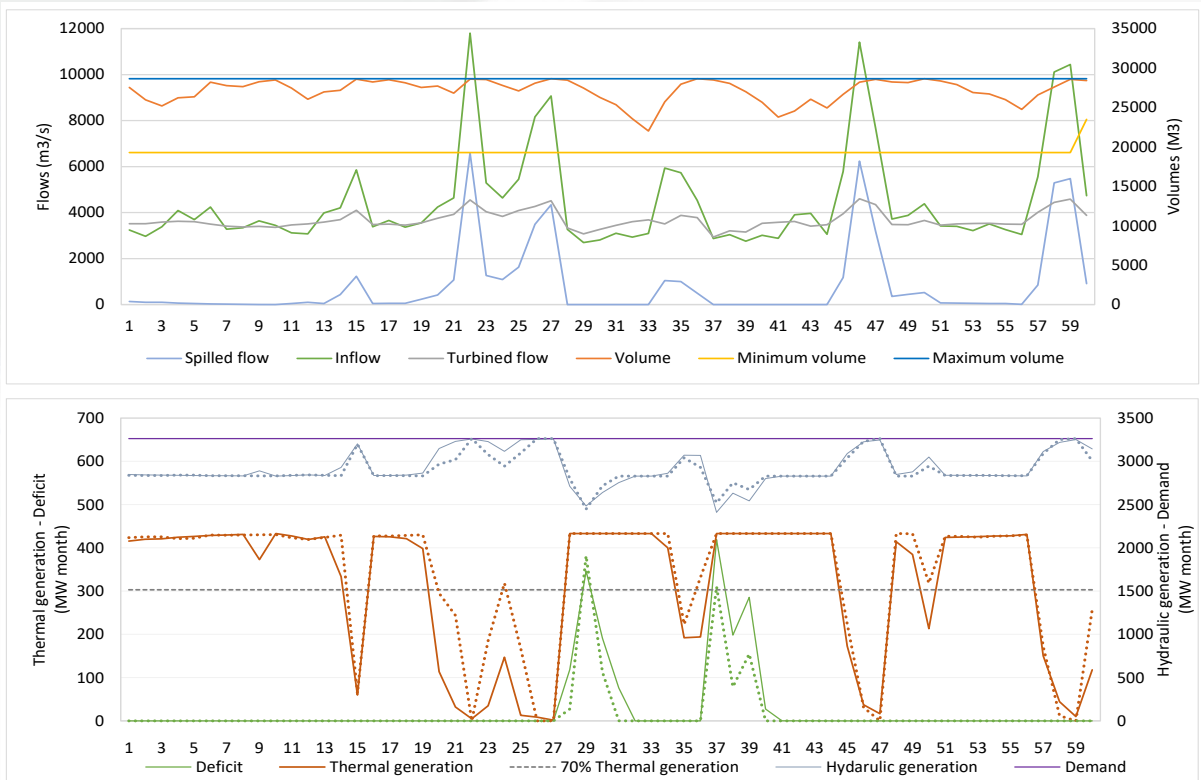

Figure 6: Results for the hydrothermal dispatch with the drought scenario – continuous lines: quadratic function; dashed lines: rational function.

Still, the rational function presented thermal generation values below 70%. This can be explained by analysing the volumes of the reservoirs, as in rainfall events occurred after a dry spell, the algorithm stores water. In those periods, the discharge cannot meet the demand. Hence, the thermal power plant is requested.

It is observed that for both functions, energy deficit occurs. The maximum deficit value for the quadratic function is 419 MW/month. With the rational function, the maximum deficit value decreased to 381 MW/month. In situations where deficits occur, they are often preceded by a decrease in inflows and accompanied by maximum thermal generation. The deficit could then be minimized by hydraulic generation, but in this situation, the optimization generates less hydraulic energy to maintain the water storage in the reservoirs.

6.2.2 Wet scenario

The optimization for the wet scenario took place between 1982 and 1986. Figure 7 shows the results obtained for the quadratic and the rational functions respectively. It can be observed that in months 6 to 41, the period in which the largest inflows occur within this scenario, the behaviour of the thermal dispatch with the quadratic function is uniformly close to zero. However, these values are not precisely zero, but rather very small energy generation orders.

Again, in real power plants, this would not be feasible, since they would indicate the operation at very low power levels. In this case, the proposed rational function was able to take these dispatches to zero, thus solving the problem.

During the 19th month, the peak streamflow was observed. It was expected that no thermal generation would occur in this period. However, this was not the case. In addition, the thermal dispatch was below the 70% mark for both the quadratic and the rational function. One of the possible causes is the increase of tailwater levels downstream of the dam due to spillage. It decreases the available head, affecting the hydraulic generation. Hence, the low-level thermal generation was required to meet the energy demand in this period.

6.2.3 Average scenario

Finally, the dispatch for an average scenario was evaluated, considering the interval from 2003 to 2007. The results obtained are shown in Figure 8. With the use of the rational function, the thermal generation had the expected behaviour, as the low-intensity operation observed with the quadratic function was improved (see months 7, 24, 25, and 28). In the months where the optimization with the quadratic function presented generation values close to zero (months 11 to 15, 17 to 21, 29 to 35, and 51 to 60), the rational function set the dispatches to zero.

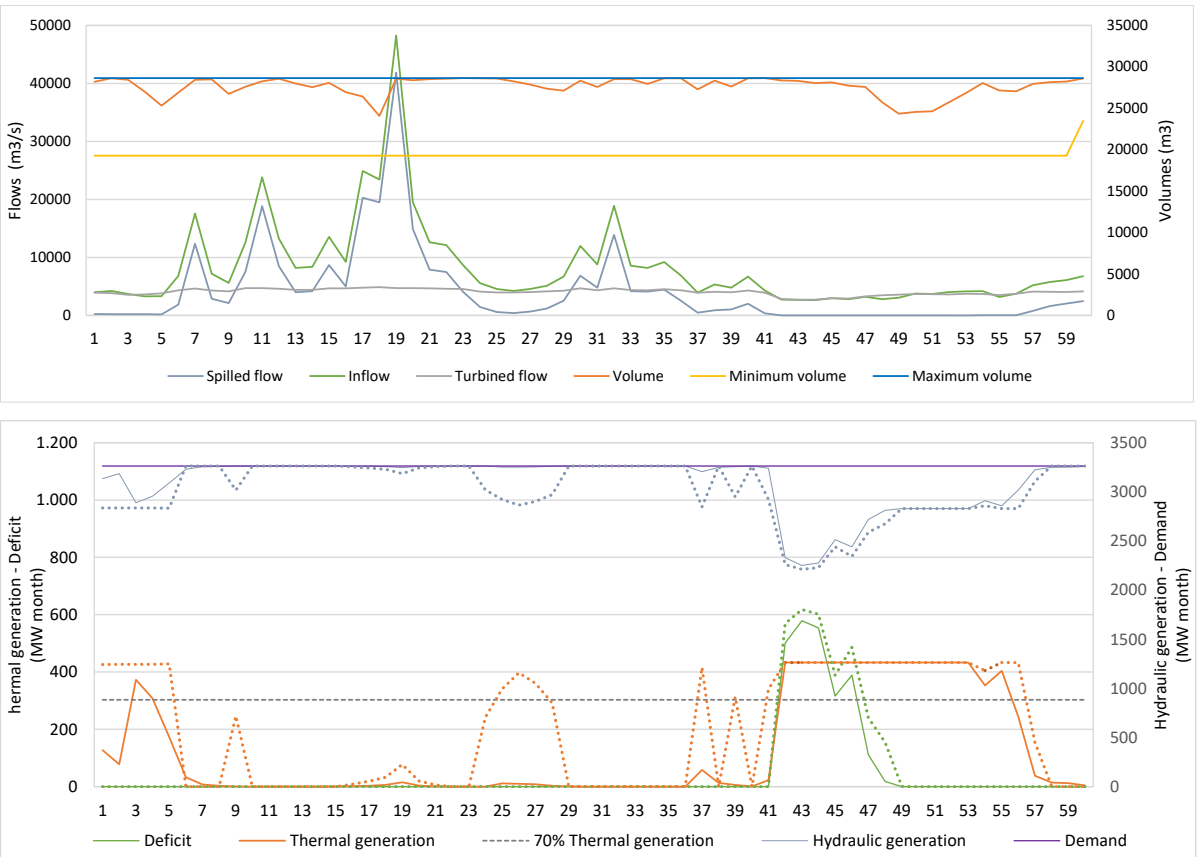

Figure 7: Results for the hydrothermal dispatch with the wet scenario – continuous lines: quadratic function; dashed lines: rational function.

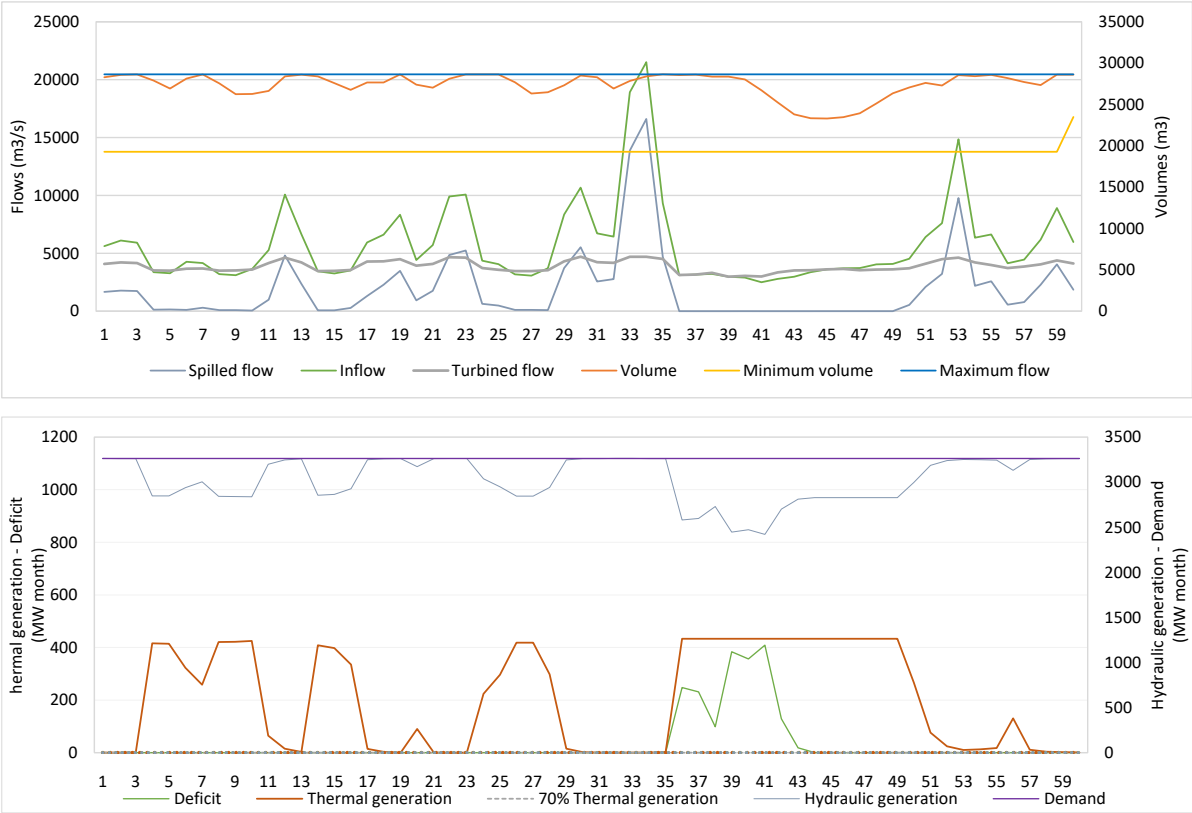

Figure 8: Results for the hydrothermal dispatch with the intermediate scenario – continuous lines: quadratic function; dashed lines: rational function.

Unlike the wet scenario, the average scenario presented occurrences of thermal generation together with spillage (months 11, 2, and 56). In such cases, it was expected the optimizer to turbine more water and minimize the thermal generation. However, this was not the case for either functions. Hence, this problem is not related to the cost functions, being specific to the optimizer.

Regarding the deficit for this scenario (months 35 to 44), it can be observed that the rational function had a higher value (3,127 MW/month) when compared to the quadratic function (1,873 MW/month). As in the previous scenarios, the inflow was not enough to meet the demand only with hydropower generation. Besides, it was not possible to use more water from the reservoir, since it would not be possible for the optimizer to restore the full storage level before the end of the period.

6.2.4 Computational aspect analysis

One of the main concerns in planning large systems is computational time, since this representation involves a large number of states, which, in turn, generates many solutions. As result, it requires a larger number of iterations to solve the problem.

The optimization algorithm was implemented in the Matlab© language. The tests were performed on a computer with Intel® Core ™ i5 processor 1.70 GHz, 4 GB RAM and 64-bit operating system.

To evaluate the model in terms of computational efficiency, the optimization time with the use of the quadratic function and the rational function was compared and shown in Figure 9. The processing time of the totality of the tests with the quadratic function was 4 hours 46 minutes, while for the rational function it was 4 hours and 14 minutes.

Table 4: Comparison of optimizations in relation to processing time.

Year	Processing time	
Drought scenario	Quadratic function (min)	Rational function (min)
1949	4.44	3.27
1950	4.57	2.81
1951	6.55	4.08
1952	3.56	3.80
1953	3.78	3.35
Wet scenario		
1982	4.17	3.95
1983	3.96	3.18
1984	3.72	3.53
1985	4.11	3.46
1986	3.03	3.39

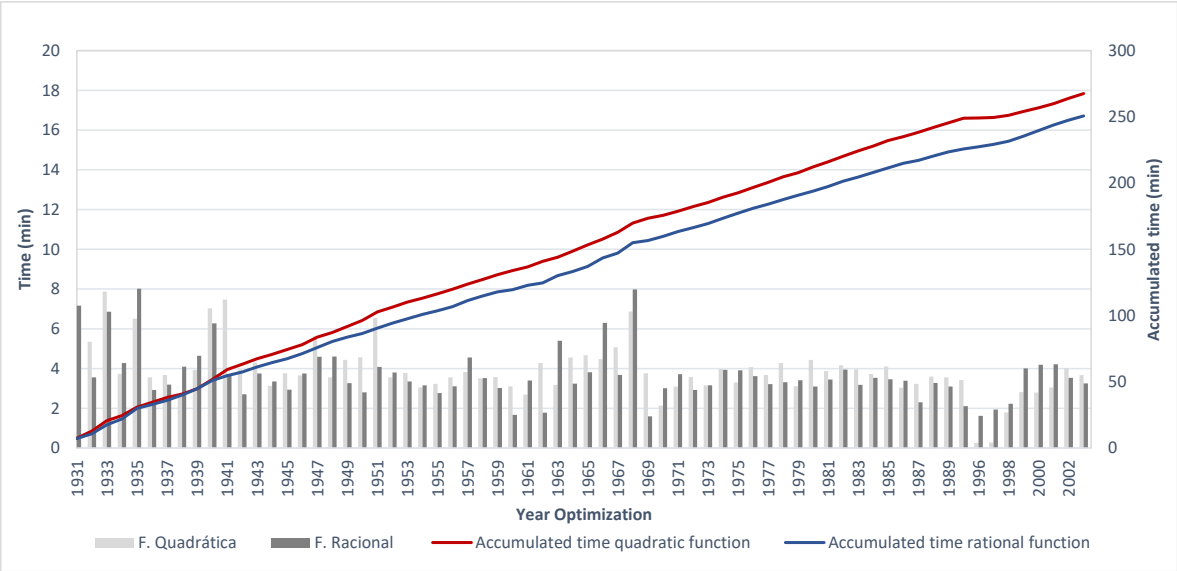

Figure 9: Computational performance with the use of quadratic and rational functions.

Table 4 shows a comparison of the processing time with the use of quadratic and rational functions for the years corresponding to drought and wet events.

The longer processing time for quadratic function can be related to the possibility of generating between 0% and 70% of the thermal power plant maximum capacity. Thus, the model takes more time deciding how to optimize in this interval. Conversely, with the rational function, it skips that region directly. This may imply significant improvements in computational performance when the optimization is performed contemplating all National Interconnected System power plants.

Conclusions

In the conventional economic dispatch problem, the cost of thermoelectric generation is represented by means of an approximation to a single quadratic function that usually does not represent the real technical, economic, and environmental characteristics of the thermoelectric units operation, since these important operating constraints are disregarded. In this way, an empirical model was presented and tested in order to obtain a cost function for a gas-fuelled thermoelectric power plant generation that is more suitable for the individualized modelling. Also, this suggested cost function is intended to be applied to nonlinear optimization models. The method penalizes low-power dispatches, to ensure the minimization of green-house effect atmospheric emissions and to represent more accurately the actual behaviour of thermal generation costs. As a result, a rational-type function was proposed.

The suggested function met the requirements of nonlinear hydrothermal dispatch optimization models since one of the concerns of using strictly non-convex functions is the existence of local minima and limited viable regions. This hinders both the estimation of the algorithm starting point and its overall convergence.

Although the proposed cost function reduced thermal dispatches in low power bands, it still could not completely avoid them. This feature is inherent to the optimization algorithm and may be linked to how the program treats the decision making of the storage as a function of the inflows. Also, dispatches in this range may be related to small amounts of energy to be generated to fully meet the demand for the period.

In an attempt to avoid the low-intensity dispatches that still occurred, penalty cost values of 10 and 20 times greater than the actual unit cost of the power plant were tested when operating at full capacity. Even so, no significant improvements were observed. For penalty values greater than 20 times the maximum power plant generation cost, the cost function loses the desired format.

A generalized characteristic in the thermal power generation behaviour when approximated to a quadratic cost function is that it operates with dispatches in very low power ranges, close to zero. In the optimization model, this characteristic would be acceptable since the costs are low. However, in a practical context, dispatches of this intensity do not resemble a real thermoelectric power plant. With the use of the rational function, the dispatches close to zero presented with the current modelling were minimized. Yet, for more detailed operating constraints (minimum generation, start-up, and shut-down costs, and environmental costs), the use of techniques such as artificial intelligence or genetic algorithms is recommended. And, of course, the operation can be refined by means of system operation simulation.

One of the main difficulties in developing this work is related to the lack of available real thermal operational and environmental costs data. Thermal power plants owner do not provide their costs for strategic reasons. Thus, approximations and theorical values were used, which limited a more in-depth analysis of the problem. Hence, further work is needed to obtain more representative

operating costs that also include thermoelectric power plants of different types of fuel. With this, a better representation of the global hydrothermal dispatch in Brazil should be possible.

As a final recommendation, the intention of this paper is not to replace the current model, but to offer an alternative that circumvents its limitations aiming at a system-level operational enhancement.

Further developments may consider more realistic simulations involving the whole Brazilian Electric System. Also, the application of a seasonal demand curve may lead to different results than the ones shown in this paper. Finally, uncertainty and risk measures could be evaluated by using synthetic streamflow series.

References

[1] CEPEL - Centro de Pesquisa de Energia Elétrica. **Modelo DECOMP: Manual de referência**, 2001. Available: http://www2.aneel.gov.br/aplicacoes/noticias/arquivos/pdf/Manual_Referencia_DECOMP.pdf [Accessed Apr. 22, 2016].

[2] SOARES, S.; CARNEIRO, A. A. F. M. Optimal Operation of Reservoirs of Electric Generation. **IEEE Transactions on Power Delivery**, v. 6, n. 3, p. 1101-1007, 1991.

[3] CEPEL - Centro de Pesquisa de Energia Elétrica, **Manual de Referência. Modelo NEWAVE**, Relatório Técnico, 2013. Available: http://www2.aneel.gov.br/aplicacoes/audiencia/arquivo/2013/086/documento/manualusuario.pdf. [Accessed: Sep. 21, 2019].

[4] PEREIRA, M. V. Optimal scheduling of hydrothermal system - an overview. IFAC **Symposium on Planning and Operation of Electric Energy Systems**, p. 1–9, 1985.

[5] PEREIRA, M. V.; PINTO, L. Optimal stochastic operation scheduling of large hydroelectric systems. International **Journal of Electrical Power and Energy Systems**, v. 11, p. 273–279, 1989.

[6] BESSA, M. R. et al. Otimização do despacho hidrotérmico mediante algoritmos híbridos com computação de alto desempenho: modelo PHOENIX. **VII Congresso de Inovação Tecnológica em Energia Eléctrica (VII CITENEL)**. Rio de Janeiro, RJ. 2013.

[7] LIU, Y.; WU, L.; LI, J. Towards accurate modeling of dynamic startup/shutdown and ramping processes of thermal units in unit commitment problems. **Energy**, v. 187, 115891,

[8] XIA, X.; ELAIW, A. Optimal Dynamic Economic Dispatch of Genaration: A review. **Electric Power Systems Research**, v. 80, p. 975-986, 2010.

[9] HUANG, C-L. et al. A new Method for Dynamic Economic Emission Dispatch Problem. **2019 IEEE Congress on Evolutionary Computation**. Wellington, New Zealand, 2019.

[10] ROY, P. K.; BHUI, S. A multi-objective hybrid evolutionary algorithm for dynamic economic emission load dispatch. **International Transactions on Electrical Energy Systems**, v. 26, p. 49-78, 2016.

[11] MASON, K.; DUGGAN, J.; HOWLEY, E. A multi-objective neural network trained with differential evolution for dynamic economic emission dispatch. **Electrical Power and Energy Systems**, v. 100, p. 201–221, 2018.

[12] MAHDI, F. P. et al. A holistic review on optimization strategies for combined economic emission dispatch problem. **Renewable and Sustainable Energy Reviews**, V. 81, p. 3006–3020, 2018.

[13] JEBARAJ, L. et al. Application of differential evolution algorithm in static and dynamic economic or emission dispatch problem: A review. **Renewable and Sustainable Energy Reviews**, v. 77, p. 1206–1220, 2017.

[14] MARCILIO, D. C. **Otimização do Depsacho Hidrotérmico Utilizando o Método de Lagrangeano Aumentado com Gradiente Espectral Projetado.** Ph.D Thesis. PPGMNE – UFPR. Curitiba, 2014.

[15] WOOD, A. J.; WOLLENBERG, B. F.; SHEBLÉ, G. F. **Power Generation, Operation and Control**. 3rd. ed. New Jersey: John Wiley & Sons, 1996.

[16] EL-HAWARY, M. E.; CHRISTENSEN, G. S. **Optimal Economic Operation of Electric Power Systems**. New York: Academic Press , 1979.

[17] SOARES, S. Planejamento da Operação de Sistemas Hidrotérmicos. **SBA: Controle de Automação**, v. 1, n. 2, p. 122-131, 1987.

[18] LEFTON, S. et al. The Increased Cost of Cycling Operation at Combined Cycle Power Plants. **International Conference on Cycling Plant**. Charlotte, North Carolina, November 2012.

[19] NAG, P. et al. Low Load Operational Flexibility for Siemens G-class Gas Turbines. **Power-Gen International**, p. 1–17, Orlando, 2008.

[20] CONAMA - CONSELHO NACIONAL DO MEIO AMBIENTE. **Resolução n. 436**, de 22 de dez. de 2011. Brasil. Avaliable: http://www.mma.gov.br/port/conama/legiabre.cfm?codlegi=520. [Acessed: Oct. 18, 2016].

[21] SEMA - Secretaria de Estado de Meio Ambiente e Recursos Hidricos. **Resolução n. 016**, de 15 de abr. de 2014. Brasil. Available: http://www.iap.pr.gov.br/pagina-934.html. [Accessed: Oct. 20, 2016].

[22] FAVORETO, R. D. E. S. Estratégias de Planejamento Empresarial: Tratamento de Incertezas de uma Empresa de Geração no Sistema Elétrico Brasileiro. **Master dissertation**. PPGERHA – UFPR. Curitiba. 2005.

[23] LABADIE, J. W. Optimal operation of multireservoir systems: state-of-the-art review. **Journal of Water Resources Planning and Management**, v. 130, n. 2, p. 93-111, 2004.